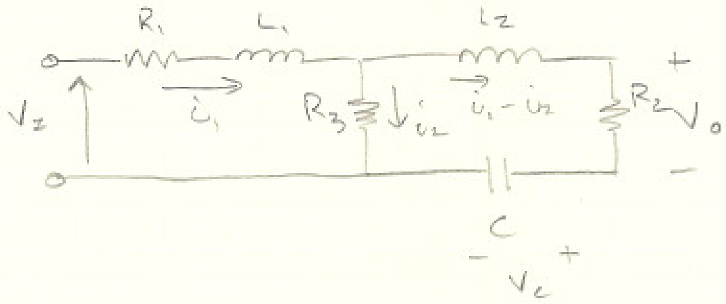


$$V_i = R_1 i_1 + L_1 \frac{di_1}{dt} + R_3 i_2 \qquad (1)$$

$$R_3 i_2 = L_2 \frac{d(i_1 - i_2)}{dt} + V_c + V_o \qquad (2)$$

$$V_c = \frac{1}{C}\int (i_1 - i_2)\,dt \Rightarrow i_1 - i_2 = C\frac{dV_c}{dt} \qquad (3)$$

$$(1) \Rightarrow \quad \frac{di_1}{dt} = -\frac{R_1}{L_1} i_1 - \frac{R_3}{L_1} i_2 + \frac{V_i}{L_1} \qquad (A)$$

$$(2) \Rightarrow \quad \frac{di_2}{dt} = -\frac{R_3}{L_2} i_2 + \frac{R_2}{L_2}(i_1 - i_2) - \frac{R_1}{L_1} i_1 - \frac{R_3}{L_1} i_2 + \frac{V_i}{L_1}$$

$$= \left(\frac{R_2}{L_2} - \frac{R_1}{L_1}\right) i_1 - \left(\frac{R_3}{L_2} + \frac{R_2}{L_2} + \frac{R_3}{L_1}\right) i_2 + \frac{V_i}{L_1} + \frac{V_c}{L_2} \qquad (B)$$

$$(3) \Rightarrow \quad \frac{dV_c}{dt} = \frac{1}{C} i_1 - \frac{1}{C} i_2 \qquad (C)$$

now we can pick our state vector.

$$\underline{x} = \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} \qquad u = V_i$$

$$V_o = y = R(i_1 - i_2)$$

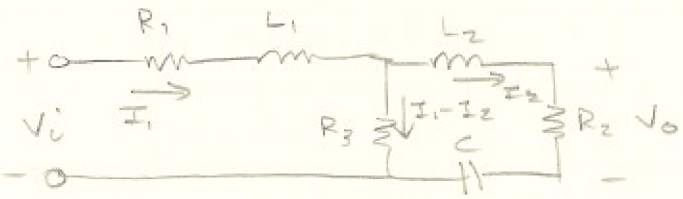
Recall

$$\underbrace{\begin{bmatrix} R_2 & -R_2 & 0 \end{bmatrix}}_{\underline{C}} \underbrace{\begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{\underline{D}} \underbrace{u}_{\underline{u}}$$

$$y = Cx + Du$$

$$\begin{bmatrix} -\frac{R_1}{L_1} & -\frac{R_3}{L_1} & 0 \\ \frac{R_2}{L_2} - \frac{R_1}{L_1} & -\frac{R_3}{L_2} - \frac{R_2}{L_2} - \frac{R_3}{L_1} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} = \underline{A} \qquad \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_1} \\ 0 \end{bmatrix} = \underline{B}$$

we know that there are an infinite number of state space repersentation, another example to show this is done by changing variables.



a new state vector is achieved.

$$z = \begin{bmatrix} I_1 \\ I_2 \\ V_c \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} -\frac{R_3+R_1}{L_1} & \frac{R_3}{L_1} & 0 \\ \frac{R_3}{L_2} & -\frac{R_3+R_2}{L_2} & -\frac{1}{L_2} \\ 0 & \frac{1}{C} & 0 \end{bmatrix} \quad \bar{C} = \begin{bmatrix} 0 & R_2 & 0 \end{bmatrix}$$

note: the eiger values from A in the first and second example are the same.

The two A matrix's can be related.

$I_1 = i_1$

$I_2 = i_1 - i_2$

$V_c = v_c$

$$\underbrace{\boxed{\begin{bmatrix} I_1 \\ I_2 \\ V_c \end{bmatrix}}}_{z} = \underbrace{\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}}_{P^{-1}} \underbrace{\boxed{\begin{bmatrix} i_1 \\ i_2 \\ v_c \end{bmatrix}}}_{x}$$

$z = P^{-1} x$

$x = P z$

$$\dot{z} = P^{-1} \dot{x} = P^{-1} \left[Ax + Bu \right]$$

$$= P^{-1} A x + P^{-1} B u$$

$$= \underbrace{\boxed{P^{-1} A P}}_{\bar{A}} z + \underbrace{\boxed{P^{-1} B}}_{\bar{B}} u$$

$$Y = Cx + Du = \underbrace{\boxed{CP}}_{\bar{C}} z + Du$$

here we see that the eigen values of $A = \bar{A}$

$$eig(\bar{A}) = eig(A)$$

$$eig(P^{-1}AP) = eig(A)$$

$$\det(\lambda I - P^{-1}AP) = 0$$

$$\det(\lambda \underbrace{\boxed{P^{-1}P}}_{I} - P^{-1}AP) = 0$$

$$\det(P^{-1}(\lambda I - A)P) = 0$$

$$\det(P^{-1})\det(P)\det(\lambda I - A) = 0$$

$$\underbrace{\boxed{\det(P^{-1}P)}}_{\det(I)=1}\det(\lambda I - A) = 0$$

$\therefore$

$$eig(\bar{A}) = eig(A)$$

note:

$$\boxed{\begin{aligned} \bar{A} &= P^{-1}AP \\ \bar{B} &= P^{-1}B \\ \bar{C} &= CP \\ \bar{D} &= D \end{aligned}}$$